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CHARGE-CONFINING GRAVITATIONAL ELECTROVACUUM SHOCK WAVE

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In previous publications, we have extensively studied spherically symmetric solutions of gravity coupled to a nonstandard type of nonlinear electrodynamics containing a square-root of the ordinary Maxwell Lagrangian (the latter is known to yield quantum chromodynamic (QCD)-like confinement in a flat spacetime). A class of these solutions describe nonstandard black holes of Reissner–Nordström–(anti-)-de Sitter type with an additional constant radial vacuum electric field, in particular, a non-asymptotically flat Reissner–Nordström-type black hole. Here, we study the ultra-relativistic boost (Lousto–Sanchez extension of Aichelburg–Sexl) limit of the latter and show that, unlike the ordinary Reissner–Nordström case, we obtain a *gravitational electrovacuum shock wave* as a result of the persistence of the gauge field due to the "square-root" Maxwell Lagrangian term. Next, we show that this gravitational electrovacuum shock wave *confines* charged test particles (both massive and massless) within a finite distance from its front.

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1. Introduction

In his analysis in Refs. 1 and 2, 't Hooft has shown that in any effective quantum gauge theory, which is able to describe linear confinement phenomena, the energy density of electrostatic field configurations should be a linear function of the electric displacement field **D** in the infrared region. In particular, a consistent quantum approach has been developed in Refs. 1 and 2, where the electric displacement field appears as an "infrared counterterm".

The simplest way to realize these ideas in flat spacetime was proposed in Refs. 3–7 by considering a nonlinear gauge theory with an action:

$$S = \int d^4x \, L(F^2) \,, \qquad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2} \,, \\ F^2 \equiv F_{\mu\nu}F^{\mu\nu} \,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,, \tag{1}$$

where f_0 is a positive coupling constant. It has been shown in Ref. 3 that the "square root" Maxwell term $\sqrt{-F^2}$ naturally arises as a result of spontaneous breakdown of scale symmetry of the original scale-invariant Maxwell action. For static field configurations the model (1) yields an energy density that indeed contains a term linear w.r.t. $|\mathbf{D}|$, where $\mathbf{D} = \mathbf{E} - \frac{f_0}{\sqrt{2}}\frac{\mathbf{E}}{|\mathbf{E}|}$. The model (1) produces a confining effective potential containing a Coulomb plus a linear one, which is of the form of the well-known "Cornell" potential[8,9] in the phenomenological description of quarkonium systems (see Ref. 4). Let us remark that one could start with the non-Abelian version of the action (1) as well, since the non-Abelian theory can effectively be reduced to an Abelian one in the static spherically symmetric case as pointed out in Ref. 4.

Coupling of the nonlinear gauge field system (1) to ordinary Einstein gravity as well as to $f(R) = R + \alpha R^2$ gravity was recently studied in Ref. 10–12,[a] where the following interesting new features of the pertinent static spherically symmetric solutions have been found:

(i) Appearance of a constant radial vacuum electric field (in addition to the Coulomb one) in charged black holes within Reissner–Nordström–(anti-)de Sitter spacetimes, in particular, in electrically neutral black holes with Schwarzschild–(anti-)de Sitter geometry. Let us particularly stress, that constant radial electric fields *do not* exist as solutions of ordinary Maxwell electrodynamics.

(ii) Novel mechanism of *dynamical generation* of cosmological constant through the nonlinear gauge field dynamics of the "square-root" gauge field term.

(iii) Even in case of vanishing effective cosmological constant the resulting Reissner–Nordström-type black hole, apart from carrying an additional constant radial vacuum electric field, turns out to be *non-asymptotically flat* — a feature resembling the gravitational effect of a hedgehog.[15,16]

(iv) New "tube-like universe" solutions of Levi–Civita–Bertotti–Robinson[17–19] type.

(v) Coupling the gravity/nonlinear gauge field system self-consistently to *light-like* branes[b] produces charge-hiding and charge-confining "thin-shell" wormhole

[a]Let us also mention the recent papers,[13,14] where coupling of ordinary Einstein gravity to the pure "square-root" gauge field Lagrangian is discussed. One of the new interesting features in this model is the existence of dyonic solutions.

[b]For a systematic reparametrization-invariant worldvolume Lagrangian formulation of *light-like* branes, see Refs. 20–25.

solutions displaying QCD-like charge confinement.[11] Similar effect is also obtained via coupling to ordinary Nambu–Goto branes.[26]

Coupling of the nonlinear gauge field system (1) to $f(R) = R + \alpha R^2$ gravity plus a dilaton[12] produces in addition to the above properties (i)–(v) also an appearance of dynamical effective gauge couplings, as well as confinement–deconfinement transition effect as functions of the dilaton vacuum expectation value. Finally, nonsingular wormhole solutions that do not require exotic matter have been found in a scalar curvature square plus Ricci squared model.[27]

One should notice that all of the above solutions are both static and spherically symmetric. The purpose of this paper is to study time-dependent "shock wave" type solutions in the gravity/nonlinear gauge field system.

Gravitational shock waves are of particular relevance (starting with the classic paper[28]) in modern field and string theory, especially due to their role in the description of impulsive light-like signals in general relativity,[29] high-energy scattering at Planck energies and ultra-relativistic heavy ion collisions (see e.g. Refs. 30–32 and references therein), etc.

Gravitational shock wave solutions can be obtained from static spherically symmetric ones by an ultra-relativistic boost procedure originating from the paper by Aichelburg and Sexl[28] (boost of ordinary Schwarzschild metric, see also Ref. 33), which was subsequently generalized to ultra-relativistic boosts of Reissner–Nordström, Reissner–Nordström–(anti-)de Sitter and Kerr–Newman geometries.[34–42]

When performing an ultra-relativistic boost of the ordinary Reissner–Nordström metric, in order to get a well-defined limit one must appropriately rescale the mass ($m \sim 1/\gamma$) and the charge ($Q^2 \sim 1/\gamma$) with the Lorentz-boost factor $\gamma = (1 - w^2)^{-\frac{1}{2}} \to \infty$ when $w \to 1$. This implies the vanishing of the electromagnetic field in the ultra-relativistic limit, although a finite δ-function contribution of the latter remains in the energy–momentum tensor, so that it contributes only to shape the gravitational shock wave profile.

The first task in this paper is to study the ultra-relativistic boost limit of the simplest nontrivial new solution obtained by coupling gravity to the nonlinear gauge field system (1), namely, the above mentioned non-asymptotically flat Reissner–Nordström type black hole carrying an additional constant radial vacuum electric field apart from the Coulomb one. Unlike ordinary Reissner–Nordström case where the electromagnetic field vanishes in the boost limit, here the resulting shock wave we obtain is a nontrivial gravitational electrovacuum one with a nonzero electric and magnetic fields originating from the constant radial vacuum electric field in the rest-frame.

The second main task is to study the dynamics of charged test particles (both massive and massless) in the newly obtained gravitational electrovacuum shock wave background. We show that the latter exhibits a QCD-like charge-confining feature.

2. Ultra-Relativistic Limit of Nonstandard Black Hole with Confining Electric Potential

The simplest coupling to gravity of the nonlinear gauge field system with a square root of the Maxwell term (1) known to produce QCD-like confinement in flat spacetime[3–7] is given by the action (we use units with Newton constant $G_N = 1$):

$$S = \int d^4x\sqrt{-g}\left[\frac{R(g) - 2\Lambda_0}{16\pi} + L(F^2)\right], \qquad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{\varepsilon F^2}\,,$$
$$F^2 \equiv F_{\kappa\lambda}F_{\mu\nu}g^{\kappa\mu}g^{\lambda\nu}\,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,, \tag{2}$$

where $R(g) = R_{\mu\nu}g^{\mu\nu}$ is the scalar curvature of the spacetime metric $g_{\mu\nu}$, $R_{\mu\nu}$ is the Ricci tensor and $g \equiv \det\|g_{\mu\nu}\|$; the sign factor $\varepsilon = \pm 1$ in the square-root term in (2) corresponds to "magnetic" or "electric" dominance — in what follows we consider the $\varepsilon = -1$ case; f_0 is a positive coupling constant measuring the strength of charge confinement.

Let us stress that we *do not* need to introduce *any* bare cosmological constant Λ_0, in (2) since the "square-root" Maxwell term dynamically generates a *nonzero effective cosmological constant* $\Lambda_{\rm eff} = 2\pi f_0^2$.[11] The role of the bare Λ_0 is just shifting the effective $\Lambda_{\rm eff}$ (see Eqs. (8) below).

The equations of motion corresponding to (2) read:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda_0\, g_{\mu\nu} = 8\pi T_{\mu\nu}\,, \tag{3}$$

$$T_{\mu\nu} = \left(1 - \frac{f_0}{\sqrt{-F^2}}\right)F_{\mu\kappa}F_{\nu\lambda}g^{\kappa\lambda} - \frac{1}{4}\left(F^2 + 2f_0\sqrt{-F^2}\right)g_{\mu\nu} \tag{4}$$

and

$$\partial_\nu\left(\sqrt{-g}\left(1 - \frac{f_0}{\sqrt{-F^2}}\right)F_{\kappa\lambda}g^{\mu\kappa}g^{\nu\lambda}\right) = 0\,. \tag{5}$$

In our preceding papers,[10,11] we have shown that the gravity/nonlinear-gauge-field system (2) possesses static spherically symmetric solutions with a radial electric field containing *both* Coulomb and *constant* "vacuum" pieces:

$$F_{0r} = \frac{\varepsilon_F f_0}{\sqrt{2}} - \frac{Q}{\sqrt{4\pi}r^2}\,, \qquad \varepsilon_F \equiv \text{sign}(F_{0r}) = -\text{sign}(Q) \tag{6}$$

and the spacetime metric:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \tag{7}$$

$$A(r) = 1 - \sqrt{8\pi}|Q|f_0 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\rm eff}}{3}r^2\,, \qquad \Lambda_{\rm eff} = \Lambda_0 + 2\pi f_0^2\,, \tag{8}$$

is that of a Reissner–Nordström–(anti-)de Sitter-type black hole depending on the sign of the *dynamically generated* effective cosmological constant $\Lambda_{\rm eff}$.

- Solutions (6)–(8) represent a *nonstandard* black hole whose "vacuum" electric field persists even in the electrically neutral case ($Q = 0$ in (6) and (8)).
- In the presence of the bare cosmological constant Λ_0, in the action (2) its only effect is shifting of the dynamically generated cosmological constant $2\pi f_0^2$ in second Eq. (8).
- Solutions (6)–(8) become a nonstandard Reissner–Nordström-type black hole with additional constant vacuum radial electric field in spite of the presence of *negative* bare cosmological constant $\Lambda_0 < 0$ with $|\Lambda_0| = 2\pi f_0^2$, i.e. $\Lambda_{\rm eff} = 0$ in (8). Another noteworthy feature is its *non-flat* spacetime asymptotics due to the "leading" constant term in the metric coefficient (8) being different from 1, when $Q \neq 0$. This resembles the effect on gravity produced by a spherically symmetric "hedgehog" configuration of a nonlinear sigma-model scalar field with $SO(3)$ symmetry (see Refs. 15 and 16).

In what follows, we will consider in more detail the latter special case — the non-asymptotically flat Reissner–Nordström-type black hole with additional constant vacuum radial electric field ($\Lambda_0 < 0$ with $|\Lambda_0| = 2\pi f_0^2$ in (2), i.e. $\Lambda_{\rm eff} = 0$ in (8)).

To derive the ultra-relativistic boost limit of (6)–(8), we follow the approach of Aichelburg–Sexl[28] and Lousto–Sanchez.[34,35] The first step is to transform to isotropic-like coordinates $(t, r) \to (t', r')$:

$$t = \frac{1}{\sqrt{c_0}} t'^{\sqrt{c_0}} , \qquad c_0 \equiv 1 - \sqrt{8\pi}|Q| f_0 , \tag{9}$$

$$r = \sqrt{c_0} r'^{\sqrt{c_0}} \left[1 + \frac{m}{2 c_0^{3/2} r'^{\sqrt{c_0}}} \right]^2 - \frac{Q^2}{4 c_0^{3/2} r'^{\sqrt{c_0}}} , \tag{10}$$

$$x' = r' \cos\varphi \sin\theta , \qquad y' = r' \sin\varphi \sin\theta , \qquad z' = r' \cos\theta ,$$

so that (6)–(8) becomes:

$$ds^2 = -\tilde{A}(t', r') dt'^2 + \tilde{B}(r')(dx'^2 + dy'^2 + dz'^2) , \tag{11}$$

$$\tilde{A}(t', r') = t'^{2(\sqrt{c_0} - 1)} A(r(r')) , \qquad \tilde{B}(r') = \frac{r^2(r')}{r'^2} , \tag{12}$$

$$F = F_{0r}\, dt \wedge dr = \left[\frac{\varepsilon_F f_0}{\sqrt{2}} - \frac{Q}{\sqrt{4\pi} r^2(r')} \right] (\tilde{A}\tilde{B})^{1/2} \frac{1}{r'} [dt' \wedge (x'\, dx' + y'\, dy' + z'\, dz')] . \tag{13}$$

The next step is to perform a Lorentz boost in e.g. x'-direction:

$$(t', x') \to (\tilde{t}, \tilde{x}) , \qquad t' = \gamma(\tilde{t} - w\tilde{x}) , \qquad x' = \gamma(\tilde{x} - w\tilde{t}) , \qquad \gamma \equiv (1 - w^2)^{-\frac{1}{2}} \tag{14}$$

and then take the ultra-relativistic limit $w \to 1$, i.e. $\gamma \to \infty$, by rescaling the black hole parameters $m = p/\gamma$, $Q^2 = q^2/\gamma$ (with p, q as finite constants) and using the distributional limits:

$$\frac{\gamma}{r'^s} \to \delta(u)\rho^{1-s}\frac{\Gamma(\frac{1}{2})\Gamma(\frac{s-1}{2})}{\Gamma(\frac{s}{2})} \quad \text{for } s > 1\,, \qquad \frac{\gamma}{r'} \to \frac{1}{|u|} - \delta(u)\ln\rho^2\,. \tag{15}$$

Here, the following notations are introduced:

$$v = \tilde{t} + \tilde{x}\,, \qquad u = \tilde{t} - \tilde{x}\,; \qquad \rho \equiv \sqrt{y_1^2 + y_2^2}\,, \qquad (y_i)_{i=1,2} \equiv \underline{y} \equiv (y', z')\,,$$
$$r' = \sqrt{x'^2 + \rho^2} \approx \sqrt{\gamma^2 u^2 + \rho^2} \quad \text{for large } \gamma\,. \tag{16}$$

The choice of the t-coordinate transformation (9) (which does not have an analog in the ordinary Reissner–Nordström case) has been made in such a way to obtain a well-defined ultra-relativistic limit for the metric (11) and (12). This limit turns out to coincide with that of the ordinary Reissner–Nordström metric:[34,35]

$$ds^2 = -dv\,du - h(\rho)\delta(u)du^2 + dy_1^2 + dy_2^2\,, \qquad h(\rho) = 8p\ln\rho + \frac{3\pi q^2}{2\rho}\,, \tag{17}$$

with nonzero Christoffel symbols:

$$\Gamma^v_{uu} = h(\rho)\delta'(u)\,, \qquad \Gamma^v_{ui} = \partial_i h\delta(u)\,, \qquad \Gamma^i_{uu} = \frac{1}{2}\partial_i h\delta(u)\,. \tag{18}$$

As in the standard case,[28,34,35] in order to remove the $\frac{1}{|u|}$ term in the g_{uu} metric component coming from second relation (15), one has to perform an additional coordinate transformation in the (v,u) subspace: $v \to v + 4p\ln|u|$, $u \to u$.

Concerning the electromagnetic field, the Coulomb piece in (13) vanishes in the ultra-relativistic limit as it happens in the ordinary Reissner–Nordström case,[34,35] however now we are left with a nontrivial limit:

$$F_{vu} = \frac{\varepsilon_F f_0}{2\sqrt{2}}\,\mathrm{sign}(u)\,, \qquad F_{ui} = y_i\frac{\varepsilon_F f_0}{\sqrt{2}}\left(\frac{1}{|u|} - \delta(u)\ln\rho^2\right). \tag{19}$$

The electromagnetic field potential corresponding to (19) is:

$$F_{vu} = -\partial_u A_v\,, \qquad A_v = -\frac{\varepsilon_F f_0}{2\sqrt{2}}|u|\,, \tag{20}$$

$$F_{ui} = -\partial_i A_u\,, \qquad A_u = \frac{\varepsilon_F f_0}{2\sqrt{2}}\rho^2\left[-\frac{1}{|u|} + \delta(u)(\ln\rho^2 - 1)\right]. \tag{21}$$

In terms of ordinary definitions of electric and magnetic fields $\mathbf{E} = (E_{\tilde{x}}, E_{y'}, E_{z'})$, $\mathbf{B} = (B_{\tilde{x}}, B_{y'}, B_{z'})$, we have:

$$E_{\tilde{x}} = \frac{\varepsilon_F f_0}{\sqrt{2}}\,\mathrm{sign}(\tilde{t} - \tilde{x})\,, \qquad B_{\tilde{x}} = 0\,,$$
$$E_{y'} = \frac{\varepsilon_F f_0}{\sqrt{2}}y'\left[\ln\rho^2\delta(\tilde{t} - \tilde{x}) - \frac{1}{|\tilde{t} - \tilde{x}|}\right] = B_{z'}\,, \tag{22}$$
$$E_{z'} = \frac{\varepsilon_F f_0}{\sqrt{2}}z'\left[\ln\rho^2\delta(\tilde{t} - \tilde{x}) - \frac{1}{|\tilde{t} - \tilde{x}|}\right] = -B_{y'}\,.$$

Let us stress that the ultra-relativistic boost limit of $F_{\mu\nu}$ obtained in (19) (or (22)) is a particular *vacuum* solution $F^2 = -f_0^2$ of the nonlinear gauge field equation (5), which *does not exist* as a solution in ordinary Maxwell electrodynamics. According to Eq. (4), its contribution to the total energy–momentum tensor is $-\frac{1}{4}f_0^2 g_{\mu\nu}$ which exactly cancels the contribution of the "bare" cosmological constant term in Einstein equation (3) where we started with ($\Lambda_0 = -2\pi f_0^2$).

Thus, the ultra-relativistic boost of the non-asymptotically flat Reissner–Nordström-type black hole with an additional constant "vacuum" radial electric field (6)–(8) describes a *gravitational electrovacuum shock wave* given by (17)–(19).

3. Charged Test-Particle Dynamics in the Electrovacuum Gravitational Shock Wave

We now consider the motion of charged test particles in the background of the gravitational electrovacuum shock wave (17)–(21). Here, we study the case of test-particle charge q_0 opposite to the pre-boost Reissner–Nordström charge Q, i.e.:

$$\mathrm{sign}(q_0) = \varepsilon_F = -\mathrm{sign}(Q)\,, \tag{23}$$

according to (6).[c] For definiteness, we take the $\mathrm{sign}(q_0) = \varepsilon_F = -\mathrm{sign}(Q) = 1$.

The pertinent point-particle action reads:

$$\begin{aligned} S_{\mathrm{particle}} &= \int d\lambda \left[\frac{1}{2e} g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} - \frac{1}{2}em_0^2 + q_0\frac{dx^\mu}{d\lambda}A_\mu\right] \\ &= \int d\lambda \left[\frac{1}{2e}\left(-\frac{dv}{d\lambda}\frac{du}{d\lambda} - h(\rho)\delta(u)\left(\frac{du}{d\lambda}\right)^2 + \frac{dy_i}{d\lambda}\frac{dy_i}{d\lambda}\right)\right. \\ &\qquad \left. -\frac{1}{2}em_0^2 + q_0\frac{dv}{d\lambda}A_v(u) + \frac{du}{d\lambda}q_0 A_u(u,\rho)\right], \end{aligned} \tag{24}$$

where λ denotes the worldline parameter, e is the worldline "einbein", m_0 is the test-particle mass, in particular, m_0 could be zero, and ρ, $h(\rho)$, A_v and A_u are given in (16), (17), (20) and (21), respectively.

Upon introducing worldline proper-time-like parameter τ with $\frac{d\tau}{d\lambda} = e$,[d] the equations of motion w.r.t. x^μ (geodesic) and e (mass-shell constraint) read:

$$\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda - q_0\dot{x}^\nu F_{\lambda\nu}g^{\lambda\mu} = 0\,, \tag{25}$$

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + m_0^2 = 0 \to -\dot{v}\dot{u} - h(\rho)\delta(u)\dot{u}^2 + \underline{\dot{y}}^2 + m_0^2 = 0\,, \tag{26}$$

where the dots represent $\frac{d}{d\tau}$ and $\Gamma^\mu_{\nu\lambda}$ and $F_{\lambda\nu}$ are given in (18) and (19).

[c] Note that the sign factor ε_F in Eqs. (19)–(22) is the only legacy of the Reissner–Nordström charge Q in the boosted electromagnetic field after performing the ultra-relativistic boost limit.
[d] This allows simultaneous treatment of both the massive and massless case; the genuine proper-time parameter in the massive case is $\bar{\tau}$ with $\frac{d\bar{\tau}}{d\lambda} = em_0$.

We have two conservation laws due to invariance of (24) w.r.t. v-translation ("light-cone energy" $\mathcal{E}$ conservation) and w.r.t. rotation in the (y_1, y_2)-subspace (angular momentum $\mathcal{J}$ conservation), which according to Noether theorem read:

$$\mathcal{E} = \frac{1}{2}\dot{u} - q_0 A_v\,, \qquad \mathcal{J} = y_1\dot{y}_2 - y_2\dot{y}_1 \equiv \underline{y} \times \dot{\underline{y}}\,, \tag{27}$$

(here $\times$ indicates two-dimensional vector product). The first "energy"-conservation Eq. (27) upon substituting the expression for A_v from (20) reads explicitly:

$$\dot{u} = -a|u| + 2\mathcal{E}\,, \qquad a \equiv \frac{q_0 f_0}{\sqrt{2}}\,, \tag{28}$$

(henceforth, we will use systematically the short-hand notation a for the constant defined in (28)).

The "mass-shell" constraint equation (26) yields a first-order equation for $v(\tau)$:

$$\dot{v} = -h(\rho)\dot{u}\delta(u) + \frac{1}{\dot{u}}(\dot{\underline{y}}^2 + m_0^2)\,, \tag{29}$$

where $y_i(\tau)$ is a solution of equation (25) for $\mu = i$ (using expression (21) for F_{ui}):

$$\ddot{y}_i + a\frac{\dot{u}}{|u|}y_i + \delta(u)\left[\frac{1}{2}\partial_i h(\rho)\dot{u}^2 - a\dot{u}y_i \ln\rho^2\right] = 0\,. \tag{30}$$

The form of the test-particle trajectory depends significantly on the initial condition for $u(\tau)$: $u_0 = u(0)$. There are three types of initial conditions:

(i) $|u_0| < 2\mathcal{E}/a$ — this is the case which will be treated below in a detailed manner. In this case, the particle trajectory "pierces" the shock wave, i.e. $u(\tau_0) = 0$ at some value τ_0 of the proper-time (worldline) parameter. We find $|u(\tau)| < 2\mathcal{E}/a$ for all τ (see Eq. (33) below) implying *confinement* (*trapping*) of the charged particle (both massive and massless) within finite distance $2\mathcal{E}/a$ from the shock wave.

(ii) $|u_0| = 2\mathcal{E}/a$ — this initial condition by consistency applies only for massless charged test particles. Now, we have $|u(\tau)| = 2\mathcal{E}/a =$ const. and $y_i(\tau) =$ const. for all τ, $v(\tau)$ is an arbitrary linear function of τ, i.e. this is free light-like motion parallel to the shock wave at a fixed "critical" distance $2\mathcal{E}/a$ from the latter.

(iii) $|u_0| > 2\mathcal{E}/a$ — in this case, $|u(\tau)| > 2\mathcal{E}/a$ for all τ (see Appendix), i.e. the particle trajectory never "pierces" the shock wave and, since now the proper-time (worldline) parameter τ flows opposite the "laboratory" time t, this motion in fact corresponds to a repulsion of an antiparticle (cf. Ref. 43).

We start with case (i). With the initial condition $|u(0)| \equiv |u_0| < 2\mathcal{E}/a$, the solution of Eq. (28) reads:

$$u(\tau) = \mathrm{sign}(\tau - \tau_0)\frac{2\mathcal{E}}{a}(1 - e^{-a|\tau-\tau_0|})\,, \tag{31}$$

where:

$$u(\tau_0) = 0\,, \qquad \tau_0 = \mathrm{sign}(u_0)\frac{1}{a}\ln\left(\frac{2\mathcal{E}/a - |u_0|}{2\mathcal{E}/a}\right). \tag{32}$$

Equation (31) implies:

$$|u(\tau)| = \frac{2\mathcal{E}}{a}(1 - e^{-a|\tau-\tau_0|}) \leq \frac{2\mathcal{E}}{a} \quad \text{for all } \tau \in (-\infty, +\infty)\,. \tag{33}$$

Henceforth, for simplicity we take $\tau_0 = 0$, i.e $u_0 = 0$.

Equation (29) yields for $v(\tau)$:

$$v(\tau) = -h(\rho_0)\theta(\tau) + \frac{1}{2\mathcal{E}}\int d\tau'\, e^{a|\tau'|}(\dot{\underline{y}}^2 + m_0^2)\,, \qquad \rho_0 = |\underline{y}(0)|\,, \tag{34}$$

whereas Eq. (30) for y_i acquire the form (taking into account (31) and using $\delta(u) = |\dot{u}|^{-1}\delta(\tau)$):

$$\ddot{\underline{y}} + \omega^2(\tau)\underline{y}(\tau) + \delta(\tau)\underline{y}(0)\left[\frac{\mathcal{E}}{\rho_0}\frac{dh}{d\rho}\bigg|_{\rho=\rho_0} - 2a\ln\rho_0\right] = 0\,, \qquad \omega^2(\tau) \equiv \frac{a^2}{e^{a|\tau|}-1}\,. \tag{35}$$

Asymptotically for large $|\tau|$, $y_i(\tau)$ is linear function:

$$\underline{y}(\tau) \approx \underline{\alpha}_{(\pm)}\tau + \underline{\beta}_{(\pm)} \quad \text{for } \tau \to \pm\infty\,, \qquad \underline{\alpha}_{(\pm)}, \underline{\beta}_{(\pm)} = \text{const.} \tag{36}$$

In the vicinity of $\tau = 0$, the solution of (35) is given in terms of Bessel functions (since $\omega^2(\tau) \approx a/|\tau|$ for $\tau \to 0$):

$$\underline{y}(\tau) = \sqrt{|\tau|}\left[\frac{1}{\sqrt{a}}\underline{c}_{(\pm)} J_1(2\sqrt{a|\tau|}) + \sqrt{a}\,\underline{d}_{(\pm)} Y_1\big(2\sqrt{a|\tau|}\big)\right], \tag{37}$$

where $\underline{c}_{(\pm)}, \underline{d}_{(\pm)} = \text{const.}$ and the subscripts $(\pm)$ correspond to $\tau > 0$ and $\tau < 0$, respectively.

Using the series expansions of the Bessel functions and requiring continuity of $\underline{y}(\tau)$ at $\tau = 0$, which implies $\underline{y}_0 \equiv \underline{y}(0) = -\frac{1}{\pi}\underline{d}_{(\pm)}$, we have for small τ (γ_0 denotes the Euler constant):

$$\underline{y}(\tau) \approx \underline{y}_0 + \tau\left[\theta(\tau)\underline{c}_{(+)} - \theta(-\tau)\underline{c}_{(-)} - \text{sign}(\tau)2a, \underline{y}_0\left(\gamma_0 - \frac{1}{2} + \frac{1}{2}\ln a|\tau|\right)\right], \tag{38}$$

$$\dot{\underline{y}}(\tau) \approx \theta(\tau)\underline{c}_{(+)} - \theta(-\tau)\underline{c}_{(-)} - \text{sign}(\tau)2a\underline{y}_0(\gamma_0 + \ln a|\tau|)\,, \tag{39}$$

$$\ddot{\underline{y}}(\tau) \approx -\underline{y}_0\frac{a}{|\tau|} + \delta(\tau)\big[\underline{c}_{(+)} + \underline{c}_{(-)} - 2a(2\gamma_0 + \ln a + \hat{c})\underline{y}_0\big], \tag{40}$$

where $\hat{c}$ is a regularization ("cut-off") dependent constant in the renormalization of the singular distribution product $\ln|\tau|\delta(\tau) = \hat{c}\,\delta(\tau)$.[e]

[e]This is done along the lines of the configuration space renormalization of ultraviolet-divergent Feynman diagrams in standard quantum field theory employing the original ideas of Stueckelberg–Peterman–Bogoliubov. They formulate renormalization of products of distributions with coinciding singularities by first defining them (upon appropriate regularization) on the configuration space with the singularity subset removed, and then extending the result to the whole space in the spirit of the general theory of distributions (for recent developments see Ref. 44 and references therein).

Substituting (38)–(40) back into (35), we obtain (recall $\rho_0 = |\underline{y}_0|$):

$$\underline{c}_{(+)} + \underline{c}_{(-)} = \underline{y}_0 \left[2a(2\gamma_0 + \ln a + \hat{c}) - \left(\frac{\mathcal{E}}{\rho_0} \frac{dh}{d\rho}\bigg|_{\rho=\rho_0} - 2a \ln \rho_0 \right) \right], \tag{41}$$

whereas substituting (36), (38) and (39) into the expression for $\mathcal{J}$ (27) yields:

$$\mathcal{J} = \underline{\beta}_{(\pm)} \times \underline{\alpha}_{(\pm)} = \pm \underline{y}_0 \times \underline{c}_{(\pm)} . \tag{42}$$

The compatibility of the last equalities in (42) follows from (41).

In ordinary boosted Schwarzschild case,[33] the transverse space coordinates $\underline{y}(\tau)$ experience a refraction (finite discontinuity of $\dot{\underline{y}}(\tau)$) at the shock wave (i.e. at $\tau = 0$). In the present case, $\underline{y}(\tau)$ experience reflection when meeting the shock wave due to the logarithmic singularity at $\tau = 0$ of $\dot{\underline{y}}(\tau)$ according to Eq. (39):

$$\dot{\underline{y}}(\tau) \approx -a\underline{y}_0 \, \mathrm{sign}(\tau) \ln|\tau| \to \pm\infty \quad \text{for } \tau \to \pm 0 . \tag{43}$$

Since the logarithmic singularity (43) is an integrable one, inserting (39) into Eq. (34), we find the same type of finite discontinuity in $v(\tau)$ at the shock wave as in the ordinary boosted Schwarzschild or Reissner–Nordström case:[33–35]

$$v(\tau) \approx v_0 - h(\rho_0)\theta(\tau) + \frac{a^2 \rho_0^2}{2\mathcal{E}} \tau (\ln|\tau|)^2 \quad \text{for small } \tau, \quad v_0 = \text{const.}, \tag{44}$$

$$[v]_0 \equiv \lim_{\epsilon \to 0} v(\epsilon) - v(-\epsilon)) = -h(\rho_0) . \tag{45}$$

Using Eq. (36) implies for large τ:

$$v(\tau) \approx \begin{cases} \frac{1}{2a\mathcal{E}} (m_0^2 + \underline{\beta}_{(+)}^2) e^{a\tau} \to +\infty , & \text{for } \tau \to +\infty , \\ -\frac{1}{2a\mathcal{E}} (m_0^2 + \underline{\beta}_{(-)}^2) e^{a|\tau|} \to -\infty , & \text{for } \tau \to -\infty . \end{cases} \tag{46}$$

Taking into account Eqs. (31), (33), (36), (38), (44) and (46) describing the test-particle trajectory, we conclude that the gravitational electrovacuum shock wave (17)–(19) *confines* charged particles with charges q_0 opposite to the pre-boost Reissner–Nordström charge Q. Namely, according to (33) $|u(\tau)| \leq \frac{2\mathcal{E}}{a}$ for all $\tau \in (-\infty, +\infty)$, and thus we deduce the size of the confining distance to be equal to $2\mathcal{E}/a \equiv 2\sqrt{2}\mathcal{E}/q_0 f_0$ for both massive and massless charged test particles.

4. Conclusion

In this paper, we studied the result of applying the Lousto–Sanchez extension of Aichelburg–Sexl ultra-relativistic boost procedure to the static spherically symmetric solutions of a nonstandard Reissner–Nordström type obtained from gravity coupled to a special kind of nonlinear gauge field system containing a "square-root" Maxwell term (2), which is known to produce charge confinement in flat space-time.

Unlike the case of the ultra-relativistic boost of ordinary Reissner–Nordström black hole solution,[34,35] where the electromagnetic field disappears in the boost limit leaving only a residual term in the energy–momentum tensor, in the present case due to the "confinement"-inducing "square-root" Maxwell term the electromagnetic field persists in the ultra-relativistic limit as a nontrivial "vacuum" solution (22) of the nonlinear electromagnetic field equations of motion — a solution, which does not exist in ordinary Maxwell electrodynamics. The resulting "boosted" solution describes a gravitational electrovacuum shock wave which possesses the remarkable property of confining (trapping) charged test particles (both massive and massless) with charges opposite to the pre-boost-limit black hole charge.

The above obtained gravitational electrovacuum shock wave plus the trapped charged particles could give us a picture of a hadron in the ultra-relativistic limit.

Another possible extension of our results would be the study of collisions of two electrovac-gravity shock waves.

Finally, a very interesting issue that deserves further attention concerns the possible processes of pair creation and of pair annihilation in the presence of the gravitational electrovacuum shock wave solution.

Appendix A

Unlike confinement of charged particles (massive and massless ones) when initially they are close enough to the shock wave ($|u_0| < 2\mathcal{E}/a$ implying $|u(\tau)| < 2\mathcal{E}/a$ for all τ), the initial condition $|u_0| > 2\mathcal{E}/a$ produces the following solution of Eq. (28) (here we take for definiteness positive u_0, the trajectory for negative u_0 is obtained by changing $(u,v) \to (-u,-v)$):

$$u(\tau) = \frac{2\mathcal{E}}{a} + \left(u_0 - \frac{2\mathcal{E}}{a}\right) e^{-a\tau}\,, \tag{A.1}$$

$$u(\tau) > \frac{2\mathcal{E}}{a} \quad \text{for all } \tau\,, \qquad u(\tau) \to \begin{cases} +\infty\,, & \tau \to -\infty\,, \\ 2\mathcal{E}/a\,, & \tau \to +\infty\,. \end{cases} \tag{A.2}$$

In particular, the trajectory never comes close to the shock wave at $u = 0$, i.e. no delta-function terms appear in (35) and (29), therefore, there are no "refraction" of $\underline{y}(\tau)$ and no discontinuity in $v(\tau)$ at $\tau = 0$. The large $|\tau|$-asymptotics of $v(\tau)$ is:

$$v(\tau) \to \begin{cases} +\infty & \text{when } \mathcal{J} \neq 0\,, \\ v_0 = \text{const.} & \text{when } \mathcal{J} = 0\,, \end{cases} \quad \text{for } \tau \to -\infty\,, \tag{A.3}$$

$$v(\tau) \to -\infty \quad \text{for } \tau \to +\infty\,, \tag{A.4}$$

where $\mathcal{J}$ indicates the conserved angular momentum. The last relations show that the proper time (worldline parameter) τ flows opposite to the "laboratory" time t, therefore, according to Ref. 43 we have to interpret the resulting trajectory given by (A.1)–(A.4) as a motion of an antiparticle which is completely repelled by the shock wave.

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